

# Elements of Problem-Centred Learning in Mathematics

SETA Talk

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07/04/22

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## **Question**

How can we address this balance so that there is more focus on (higher-order) graduate skills such as problem-solving and critical thinking?

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## Relation to Problem-Based Learning (PBL)

PCL can be thought of a looser version of PBL.

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- meta-cognitive and reflective skills [Hmelo-Silver, 2004].

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- meta-cognitive and reflective skills [Hmelo-Silver, 2004].
- long-term memory [Yew and Goh, 2016].

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## **Caveat**

Students sometimes just need facts first to develop certain skills [Willingham, 2009].

## Description:

1. Present a problem at the beginning of the lecture as a *starter*.  
In fact, this would be up on the board before as they arrived.
2. Give the students a moment to work on the problem (individually or in groups - as they pleased).
3. Discuss the problem as a class.
4. From the discussion draw out the objectives and desired outcomes for the session.

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- *Dialogic teaching* [Alexander, 2008] techniques are key part of the class discussion.
- I generally do not insist that students work on the problems in groups (for practical reasons).

# Units

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## 0B2 2018/19:

Class Size	Lectures	Tutorials	Assessments
~330	2 per week	1 per week	2 in-class tests (10% each), an exam at the end (80%).

## Example 1

### Exercise (1)

*Suppose that the population of a country is increasing at a rate of 20,000 a year. Express the population  $P(t)$  in year  $t$  as a function of  $t$ .*

### Exercise (2)

*In another country the rate of increase is equal to 0.2 times the current population. Can you express the population  $P(t)$  in year  $t$  as a function of  $t$ ?*

### Exercise (3)

*In a third country we have:*

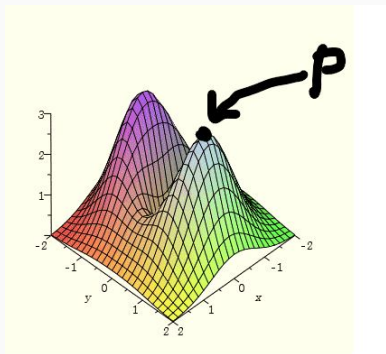
- 1. the birth rate at time  $t$  is equal to 0.03 times the population at that point.*
- 2. the death rate at time  $t$  is equal to 0.01 times the population at that point.*

*Can you express/model the population  $P(t)$  in year  $t$  as a function of  $t$ ?*

## Example 2

### Exercise

Consider the following surface:

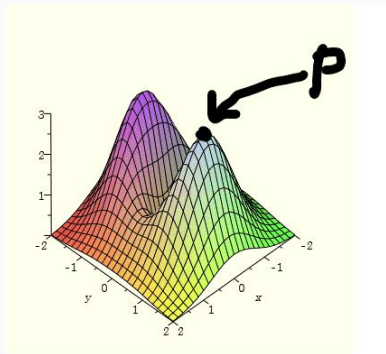


1. What can you say about the point  $P$  and why?

## Example 2

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Consider the following surface:



1. What can you say about the point  $P$  and why?
2. Given that the surface above is the graph of  $f(x, y) = (x^2 + 3y^2)\exp(1 - x^2 - y^2)$ , can you find out what  $P$  actually is?

## Example 3

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Consider the ODE

$$\frac{dy}{dx} = \frac{xy - y^2}{x^2 + xy}.$$

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Since  $y$  is a function in  $x$ , so is  $v$ . For example, if  $y = y(x) = x^2$  then  $v = v(x) = y/x = x$ .





## Feedback from Peers

- “I liked that you had a warm-up problem on the slides for students to engage with as they were getting ready for lecture. This led onto a recap of what happened last time, and discussion (and engagement from students) as to where today’s lecture is heading.”

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- “The student involvement was great - good ideas were suggested”
- “This was the right place to outline the objective and aims as they were motivated by the question. They were stated clearly, in relation to the context of the course.”

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- “He is very good in engaging student to participate in the lecture.”
- “Student engagement, asking questions, encouraging students to try to figure things out themselves” .
- “I liked how the lectures were structured, always starting with a Starter...”

Feedback specifically about these “starters” is limited.



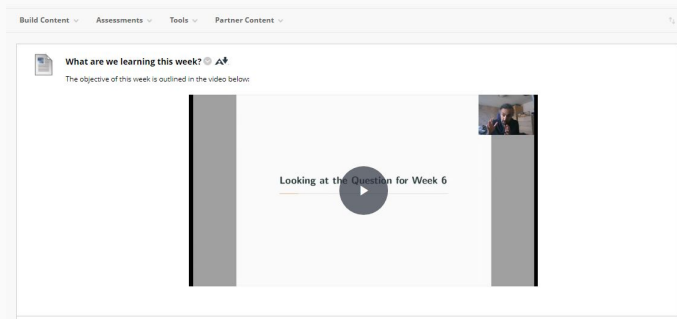
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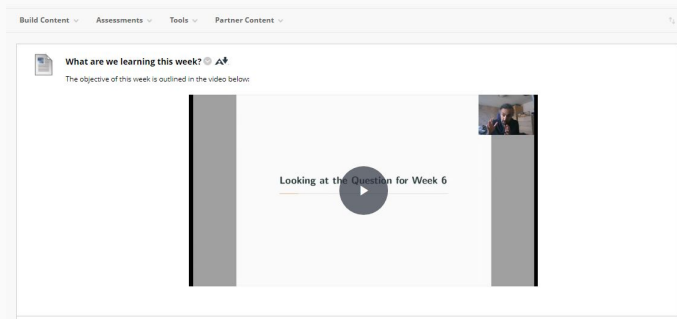


The screenshot shows a user interface for a learning management system. At the top, there are navigation tabs: "Build Content", "Assessments", "Tools", and "Partner Content". Below these is a main content area with a header "What are we learning this week?" and a sub-header "The objective of this week is outlined in the video below:". The central element is a video player with a large play button in the center. The text "Looking at the Question for Week 6" is overlaid on the play button. A small video thumbnail in the top right corner of the player shows a person speaking.

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**Does it work?** I do not think so. No mention of it in the feedback and I am not sure many students I watching them.

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Any ideas?



Alexander, R. (2008).

**Towards Dialogic Teaching: Rethinking Classroom Talk.**

Dorchester Publishing Company, Incorporated.



Asad, M., Iqbal, K., and Sabir, M. (2015).

**Effectiveness of problem based learning as a strategy to foster problem solving and critical reasoning skills among medical students.**

*Journal of Ayub Medical College, Abbottabad : JAMC*,  
27(3):604—607.



Cook, K. E., Han, Y.-L., Shuman, T. R., and Mason, G. (2017).

**Effects of integrating authentic engineering problem centered learning on student problem solving.**

*International Journal of Engineering Education*, 33(1):272–282.



Galand, B. and Frenay, M. (2005).

**L'approche par problèmes et par projets dans l'enseignement supérieur: Impact, enjeux et défis.**

Presses univ. de Louvain.



Hmelo, C. E. D. (2000).

**Problem-Based Learning: Gaining Insights on Learning Interactions Through Multiple Methods of Inquiry, pages 1–16.**

Lawrence Erlbaum Associates, Inc., Mahwah, New Jersey.



Hmelo-Silver, C. E. (2004).

**Problem-based learning: What and how do students learn?**

*Educational psychology review*, 16(3):235–266.



Jonassen, D. H. and Hung, W. (2012).

**Problem-Based Learning, pages 2687–2690.**

Springer US, Boston, MA.



Willingham, D. (2009).

**Why don't children like school?: a cognitive scientist answers questions about how the mind works and what it means for your classroom.**

Jossey-Bass.



Wu, H. and Hsieh, C. (2006).

**Developing sixth graders' inquiry skills to construct explanations in inquiry-based learning environments.**

*International Journal of Science Education*, 28(11):1289–1313.



Yew, E. H. and Goh, K. (2016).

**Problem-based learning: An overview of its process and impact on learning.**

*Health Professions Education*, 2(2):75–79.